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## Optimum design of laminated composite foam-filled sandwich plates subjected to strength constraint

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### Abstract

Optimum design of laminated composite sandwich plates with both continuous (core thickness) and discrete (layer group fiber angles and thicknesses) design variables subjected to strength constraint is studied via a two-level optimization technique. The strength of a sandwich plate is determined in a failure analysis using the Tsai–Wu failure criterion and the finite element method which is formulated on the basis of the layerwise linear displacement theory. In the first level optimization of the design process, the discrete design variables are temporarily treated as continuous variables and the corresponding minimum weight of the sandwich plate is evaluated subject to the strength constraint using a constrained multi-start global optimization method. In the second level optimization, the optimal solution obtained in the first level optimization is used in the branch and bound method for solving a discrete optimization problem to determine the optimal design parameters and the final weight of the plate. Failure test of laminated composite foam-filled sandwich plates with different lamination arrangements are performed to validate the proposed optimal design method. A number of examples of the design of laminated composite foam-filled sandwich plates are given to demonstrate the feasibility and applications of the proposed method. © 1999 Elsevier Science Ltd. All rights reserved.

*Keywords:* Composite materials; Sandwich plates; Laminates; Optimization; Finite element method; First-ply failure; Stress analysis

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### 1. Introduction

Due to their many superior mechanical properties, laminated composite sandwich plates have recently found broad applications in the aerospace and automotive industries. In general, the structures fabricated with laminated composite sandwich plates are weight sensitive and reliability stringent. The design of light weight but reliable laminated composite sandwich plates has thus become an important topic of research. In the past, many researchers have investigated the

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mechanical behaviors of sandwich plates and a number of techniques have been proposed for sandwich plate analysis (e.g., Pagano, 1970; Whitney, 1972; Pandya and Kant, 1988; Rao and Meyer-Piening, 1991). Recently, a number of researchers have studied the optimal design of laminated composite plate/sandwich structures (Vinson and Handel, 1988; Bushnell, 1990; Park, 1982; Olsen and Vanderplaats, 1989; Kodiyalam et al., 1996). For instance, Bushnell (1990) used conventional numerical optimization techniques for optimal design of truss-core sandwich structures. Olsen and Vanderplaats (1989) presented a nonlinear discrete optimization method for minimum weight design of composite sandwich panels. It is noted that the optimal design algorithms presented by Bushnell, and Olsen and Vanderplaats can only yield a single local optimum solution and sometimes may even have difficulty in searching for the optimum. Kodiyalam et al. (1996) used the genetic search method for optimal design of composite sandwich structures with discrete design variables. It has been shown that genetic algorithms may have the potential to become a viable tool for dealing with discrete optimization problems and the ability of yielding multiple optima. Despite the successful use of genetic search algorithms in the discrete optimization of composites, a major drawback of this search technique is that it often requires a prohibitively high amount of computational time for finite element-based structural optimization problems. Meaquita and Kamat (1987) used the branch and bound method (Dakin, 1965) to study the optimal design of stiffened laminated composite plates with frequency constraints in which the optimization problem was formulated as a nonlinear mixed integer programming problem. Although, in the past decade, a number of techniques have been developed for the optimal design of laminated composite structures with discrete design variables, rigorous and practical mathematical methods for solving nonlinear mixed-discrete optimization problems are not readily available and more research work is still required.

Recently, Kam and associates have studied the global optimal design of laminated composite plates (Kam et al., 1996; Kam and Chang, 1992; Kam and Lai, 1995; Kam and Snyman, 1991). In their study, layer thicknesses and fiber angles were treated as continuous design variables and their global optimal values were determined using a multi-start global optimization technique. In this paper, the previously proposed global optimization method is incorporated into the branch and bound method to formulate a two level optimization technique for conducting the minimum weight design of thick laminated composite sandwich plates with continuous and discrete design variables subject to strength constraint. The efficiency of the proposed method in yielding the global optimum is investigated and the factors that have important effects on the optimal design parameters and weights of the sandwich plates are identified via a number of examples. The accuracy of the method in obtaining the global optimal design of laminated composite sandwich plates is also verified by experimental results. Finally, the applications of the optimal design method are demonstrated by means of a number of examples of the design of laminated composite sandwich plates with different loading conditions, aspect ratios, numbers of layer groups and boundary conditions.

## **2. Strength analysis of laminated composite sandwich plate**

The laminated composite sandwich plate in Fig. 1 is composed of a homogeneous core and two laminated composite cover sheets. The  $x$  and  $y$  coordinates of the plate are taken in the midplane

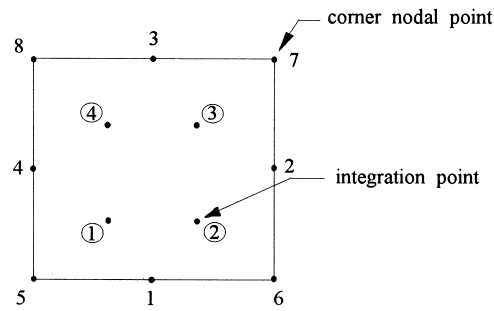
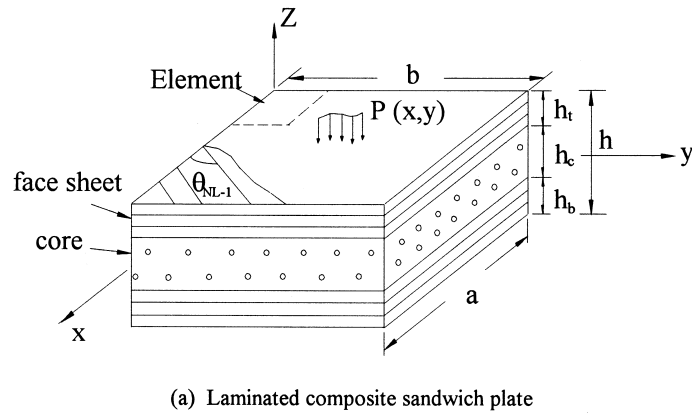


Fig. 1. Geometry and loading conditions of a laminated composite sandwich plate.

of the core which has  $a \times b$  and thickness  $h_c$ . The previously proposed finite element method (Kam and Jan, 1995) which was formulated on the basis of a layerwise linear displacement theory will be extended to the sandwich plate analysis. Herein, the sandwich plate is divided into a number of mathematical layer groups across the plate thickness and the displacement components of each layer group are assumed to vary linearly. Each layer group may contain a number of plies which have the same fiber angles. The core of the plate is treated as a layer group in the following formulation. If necessary, the core can also be divided into a number of layer groups. A brief description of the method is given as follows:

$$\mathbf{u}_1 = \mathbf{u}_o + \xi_1 \psi_1$$

and

$$\mathbf{u}_i = \mathbf{u}_o + \frac{t_1}{2} \psi_1 + \sum_{k=2,4,\dots}^{i-2} t_k \psi_k + \xi_i \psi_i \quad (i = 2, 4, 6, \dots, NP - 1)$$

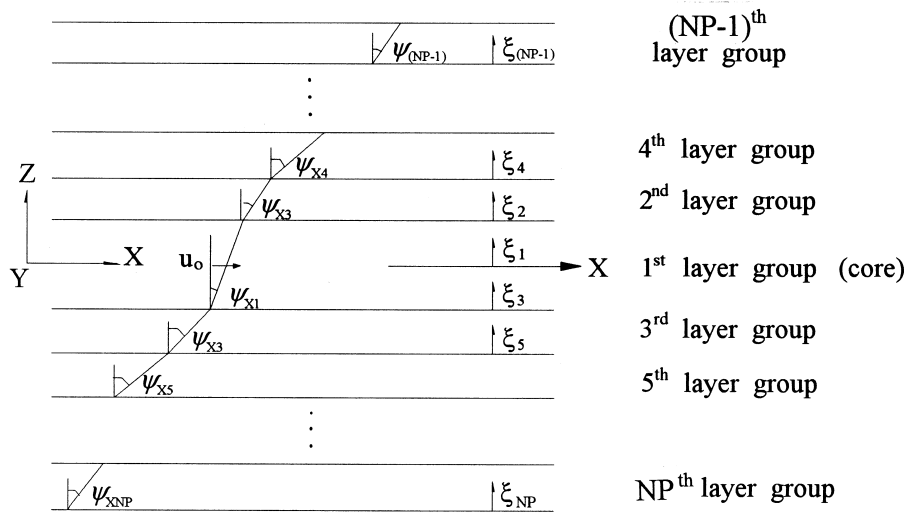


Fig. 2. Layerwise displacement components and local coordinates of layer groups.

$$u_i = u_o - \frac{t_1}{2} \psi_1 - \sum_{k=3,5,\dots}^{i-2} t_k \psi_k + \xi_i \psi_i \quad (i = 3, 5, 7, \dots, NP) \tag{1}$$

where  $u_i = (u, v, w)^t$  is the vector of displacements of the  $i$ th layer group;  $\xi_i$  is the local coordinate for the  $i$ th layer group;  $u_o$  is the displacement vector in the mid-plane;  $\psi_i = (\psi_{xi}, \psi_{yi}, \psi_{zi})^t$  is the vector of rotational degrees of freedom of the  $i$ th layer group;  $NP$  is number of layer groups;  $t_i$  is the thickness of the  $i$ th layer group. It is noted that no summation is performed in the above equations if  $(i-2)$  is less than  $k$ . Figure 2 shows the positive directions of  $\xi_i$  and  $\psi_{xi}$  together with the displacements of the layer groups across the plate thickness in the  $x$ -direction.

Three dimensional stress–strain and strain–displacement relations of the linear elasticity theory are adopted in deriving the finite element on the basis of the principle of minimum total potential energy. In the finite element formulation, the plate is discretized into a number of elements which are connected together via the nodes at the layer groups of the elements. The displacements at any point in each layer group are obtained via the method of interpolation using the layer nodal displacements and appropriate shape functions.

$$u_o = N \bar{u} \quad \psi_i = N \bar{\psi}_i \quad i = 1, 2, 3, \dots, NP \tag{2}$$

where  $N$  is a  $3 \times ND$  shape function matrix;  $(\bar{\tau})$  denotes layer group nodal displacement vector;  $ND$  is number of layer group nodes. Herein, the element stiffness matrix is constructed using a quadratic ( $ND = 8$ ) formulation of the serendipity family in which the numerical integration schemes with  $2 \times 2$  and  $3 \times 3$  Gauss rules are used for layer rotations in orthotropically or generally laminated composite face sheets, respectively. As for the sandwich core,  $3 \times 3$  and  $2 \times 2$  Gauss rules are used for mid-plane displacements and layer rotations, respectively. In the stress analysis, the stresses at the four corner nodal points in Fig. 1b are obtained from those at the integration points using the following extrapolation equations,

$$\begin{aligned} \sigma_{i+5} &= \left(1 + \frac{\sqrt{3}}{2}\right)\sigma_{i+1} + \left(1 - \frac{\sqrt{3}}{2}\right)\sigma_{3-i} - 0.5(\sigma_2 + \sigma_4) \quad i = 0, 2 \\ \sigma_{i-6} &= \left(1 + \frac{\sqrt{3}}{2}\right)\sigma_{i+2} + \left(1 - \frac{\sqrt{3}}{2}\right)\sigma_{4-i} - 0.5(\sigma_1 + \sigma_3) \quad i = 0, 2 \end{aligned} \tag{3}$$

where  $\sigma_i$  ( $i = 1, \dots, 4$ ) are stresses at integration points;  $\sigma_i$  ( $i = 5, \dots, 8$ ) are the corresponding stresses at the corner points. The stresses obtained in the finite element analysis are then used to evaluate the failure load  $P_c$  of the laminated composite plate on the basis of the Tsai–Wu criterion (Tsai and Wu, 1971). It is noted that initial failure may occur in the face sheets or the core depending on the stress states in those components and the load that causes initial failure in the sandwich plate is defined as the failure load of the sandwich plate. Other failure modes such as plate buckling, face wrinkling, etc., which can be easily included in the calculation of the failure load  $P_c$ , will not be considered in the failure analysis of the laminated composite sandwich plates. Hereafter, without loss of generality, the failure load is treated as the strength of the sandwich plate in the following optimal design.

### 3. Optimal design of sandwich plates

Consider the optimal design of a laminated composite sandwich plate composed of  $NL$  layer groups subjected to the applied load  $Pf(x, y)$  where  $P$  is the amplitude and  $f(x, y)$  the shape function. The objective is to select the optimal fiber angles and thicknesses of the layer groups in the cover sheets and the core thickness which yield the minimum weight of the sandwich plate for a given required strength. Herein, the fiber angles and thicknesses of layer groups in cover sheets can only take on discrete values while core thickness is treated as a continuous design variable. In mathematical form, the discrete optimum design problem is stated as

$$\begin{aligned} \text{minimize } W(\mathbf{h}, \boldsymbol{\theta}) &= \sum_{i=1}^{NL-1} \rho Ah_i + \rho_c Ah_c \\ \text{subject to } 0 \leq \theta_i \leq \pi \quad \theta_i &= m_i \theta_o \quad h_i \geq 0 \quad i = 1, \dots, NI-1 \\ I-1 \quad h_i &= n_i h_o \quad P_c \geq P \quad h_c \geq 0 \quad h \leq h^* \end{aligned} \tag{4}$$

where  $\rho_c$ ,  $h_c$  are density and thickness of core, respectively;  $\rho$  is density of cover laminates;  $\boldsymbol{\theta} = (\theta_1, \theta_2, \dots, \theta_{NI-1})^t$ ,  $\mathbf{h} = (h_1, h_2, \dots, h_{NI-1}, h_c)^t$  are vectors of fiber angles and thicknesses of layer groups, respectively;  $\theta_o$  and  $h_o$  are production constants;  $h^*$  is the maximum allowable plate thickness;  $W$  is plate weight;  $NI$  is either equal to  $NL$  for generally laminated composite sandwich plates or  $(NL + 1)/2$  for symmetrically and anti-symmetrically laminated composite sandwich plates;  $m_i$ ,  $n_i$  are positive integers to be determined. It is noted that the strength as well as the weight of the plate are implicitly dependent on the fiber angles. The core of the sandwich plate is treated as a layer group and its density a constant. Foam density should be treated as a design variable if optimal dynamic characteristics of sandwich plates are desired. In the present study, however, foam density has no effect on the optimal solutions of the laminated composite sandwich

plates when subjected to strength constraint. The number of layer groups  $NL$  and layer group thicknesses  $h_i$  may be different from those  $(NP, t_i)$  adopted in the finite element analysis.

The solution of the above constrained discrete minimization problem will be accomplished using the approach of two-level optimization. In the first level optimization, the layer group parameters are temporarily treated as continuous design variables and a constrained multi-start global minimization method is used to solve the above minimization problem to attain the transitional global minimum. In the second level optimization, the transitional global optimal solution is used as the starting point in the branch and bound method for determining the true global optimal solution of the problem.

### 3.1. First level optimization

The above problem of eqn (4) is first converted into an unconstrained minimization problem by creating the following general augmented Lagrangian

$$\Psi(\boldsymbol{\theta}, \mathbf{h}, \boldsymbol{\mu}, \eta, r_p) = W(\boldsymbol{\theta}, \mathbf{h}) + \sum_{j=1}^{NI} [\mu_j x_j + r_p x_j^2] + \sum_{i=1}^2 [\eta_i \phi_i + r_p \phi_i^2] \quad (5)$$

with

$$x_j = \max \left[ g_j(h_j), \frac{-\mu_j}{2r_p} \right] \quad g_j(h_j) = -h_j \leq 0 \quad j = 1, \dots, NI-1 \quad g_j(h_j) = -h_c \leq 0 \quad j = NI$$

$$\phi_i = \max \left[ H_i(\boldsymbol{\theta}, \mathbf{h}), \frac{-\eta_i}{2r_p} \right] \quad i = 1, 2 \quad H_1(\mathbf{h}, \boldsymbol{\theta}) = P - P_c \leq 0 \quad H_2(\mathbf{h}, \boldsymbol{\theta}) = h - h^* \leq 0 \quad (6)$$

where  $\mu_j, \eta_i, r_p$  are multipliers;  $\max[*,*]$  takes on the maximum value of the numbers in the bracket. Herein, the design parameters  $(\boldsymbol{\theta}, \mathbf{h})$  are treated as continuous variables and equality constraints in eqn (4) are disregarded in this level of optimization. The update formulas for the multipliers  $\mu_j, \eta_i$  and  $r_p$  are

$$\mu_j^{n+1} = \mu_j^n + 2r_p^n x_j^n \quad j = 1, \dots, NI$$

$$\eta_i^{n+1} = \eta_i^n + 2r_p^n \phi_i^n$$

$$r_p^{n+1} = \begin{cases} \gamma_o r_p^n & \text{if } r_p^{n+1} < r_p^{\max} \\ r_p^{\max} & \text{if } r_p^{n+1} \geq r_p^{\max} \end{cases} \quad (7)$$

where the superscript  $n$  denotes iteration number;  $\gamma_o$  is a constant;  $r_p^{\max}$  is the maximum value of  $r_p$ . The parameters  $\mu_j^o, \eta_i^o, r_p^o, \gamma$  and  $r_p^{\max}$  must be determined by the method of trial and error. From experience, the initial values of the multipliers and the values of the parameters  $(\gamma_o, r_p^{\max})$  are chosen as

$$\mu_j^o = 1.0 \quad j = 1, \dots, NI \quad \eta_i^o = 1.0 \quad r_p^o = 0.4 \quad \gamma = 1.25 \quad r_p^{\max} = 100 \quad (8)$$

The minimum weight design problem of eqn (5) has thus become the solution of the following unconstrained optimization problem.

$$\begin{aligned} & \text{Minimize } \Psi(\boldsymbol{\theta}, \mathbf{h}, \boldsymbol{\mu}, \boldsymbol{\eta}, r_p) \quad \text{with respect to } \boldsymbol{\theta} \text{ and } \mathbf{h} \\ & \text{subject to } 0 \leq \theta_i \leq \pi \quad i = 1, \dots, NI-1 \end{aligned} \quad (9)$$

The solution of the above unconstrained optimization problem is straightforward by using the previously proposed unconstrained multi-start global optimization algorithm (Kam and Snyman, 1991; Snyman and Fatti, 1987). In general, around eight to ten starting points and six to eight iterations for each starting point are required to obtain the global optimum. It is noted that if the solution obtained at this level of optimization satisfies all the constraints in eqn (4), it is then treated as the true global optimal solution of the problem and the second level optimization is disregarded.

### 3.2. Second level optimization

Once the transitional global optimal solution of the first level optimization has been obtained, the possible values of multipliers  $m_i$  for layer group fiber angles are determined as

$$m_i = \left\{ \left[ \frac{\theta_i}{\theta_o} \right], \left[ \frac{\theta_i}{\theta_o} \right] + 1 \right\} \quad i = 1, \dots, NI-1 \quad (10)$$

where  $[\cdot]$  denotes the integral part of the number in the bracket. In the present study, it has been found that for small  $\theta_o$ , say  $\theta_o \leq 10^\circ$ , the design space is so flat with respect to fiber angles in the vicinity of the optimum that small perturbations in  $\theta_i$  do not affect the final optimum plate weight. The final fiber angles can then be determined immediately using the method of rounding in which the fiber angles obtained in the first level optimization are rounded off to the nearest available discrete values given in eqn (10). After the above modification, the transitional global optimal solution used for this level of optimization can be rewritten as  $(h_1, h_2, \dots, h_{NI-1}, h_c)$  and is termed modified transitional global optimum. The optimization problem at this level then becomes the determination of the ply numbers of the layer groups of the cover sheets,  $n_i$ , and core thickness  $h_c$  that will minimize the plate weight and also satisfy the constraints in eqn (4).

$$\begin{aligned} & \text{Minimize } W(\mathbf{n}, h_c) \quad \mathbf{n} = (n_1, n_2, \dots, n_{NI-1}) \\ & \text{Subject to } P_c \geq P \quad n_i \geq 0 \\ & \quad h_i = n_i h_o \quad i = 1, \dots, NI-1 \quad h_c \geq 0 \quad h \leq h^* \end{aligned} \quad (11)$$

The above nonlinear mixed integer programming problem is then solved by Dakin's branch and bound method which has been proved to be well suited to solve large nonlinear mixed integer programming problems (Gupta and Ravindran, 1983). Herein, Dakin's algorithm for the solution of the above problem starts with the modified transitional global optimal solution at node 1. If the integer variable of ply number  $n_k$  is nonintegral and takes on the value  $h_k/h_o$ , it is then divided into integral and fractional parts, i.e.,

$\left[ \frac{h_k}{h_o} \right]$  and  $f_k$ , respectively, as

$$n_k = \left[ \frac{h_k}{h_o} \right] + f_k \quad (12)$$

Since  $n_k$  must be an integer, either one of the following two conditions must be satisfied

$$n_k \leq \left[ \frac{h_k}{h_o} \right] \quad (13)$$

or

$$n_k \geq \left[ \frac{h_k}{h_o} \right] + 1 \quad (14)$$

the above conditions make node 1 branching into two subproblems which belong to two different nodes, say, nodes 2 and 3.

Subproblem A (node 2):

$$\begin{aligned} &\text{Minimize } W(\mathbf{n}, h_c) \quad \mathbf{n} = (n_1, n_2, \dots, n_{NI-1}) \\ &\text{Subject to } P_c \geq P \quad n_i \geq 0 \quad i = 1, \dots, NI-1 \\ &\quad \quad \quad n_k \leq \left[ \frac{h_k}{h_o} \right] \quad h_c \geq 0 \quad h \leq h^* \end{aligned} \quad (15)$$

Subproblem B (node 3):

$$\begin{aligned} &\text{Minimize } W(\mathbf{n}, h_c) \quad \mathbf{n} = (n_1, n_2, \dots, n_{NI-1}) \\ &\text{Subject to } P_c \geq P \quad n_i \leq 0 \quad i = 1, \dots, NI-1 \\ &\quad \quad \quad n_k \leq \left[ \frac{h_k}{h_o} \right] + 1 \quad h_c \geq 0 \quad h \leq h^* \end{aligned} \quad (16)$$

In the above subproblems, the integrality requirement on the design variables has been removed. Each of these subproblems are solved by utilizing the forementioned constrained global optimization algorithm in which the modified transitional global optimal solution is used as the single starting point. In general, around three iterations are required to find the optimal solution for each subproblem. The above branching process is repeated until all nodes are fathomed and the optimal solution to the given discrete optimization problem is obtained. It is worth noting that the use of any of the local optima obtained in the first level optimization as the starting point in the second level optimization will produce a solution more inferior than that obtained via the above procedure.



#### 4. Experimental investigation

Five laminated composite foam-filled sandwich plates ( $a = 10$  cm) of different layups and core thicknesses, namely,  $[0_3^{\circ}/0.142 \text{ cm}]_s$ ,  $[0^{\circ}/90^{\circ}/0^{\circ}/0.144 \text{ cm}]_s$ ,  $[45^{\circ}/-45_2^{\circ}/0.136 \text{ cm}]_s$ ,  $[45_2^{\circ}/-45_3^{\circ}/0.422 \text{ cm}]_s$  and  $[45^{\circ}/-45_2^{\circ}/-45_2^{\circ}/0.420 \text{ cm}]_s$ , were manufactured and tested to failure. The cover laminates were made of graphite/epoxy (Q-1115) prepreg tape supplied by the Toho Co., Japan and the core was @410-1 C. S. polyurethane foam tape imported from the U.S.A. The on-axis properties of the cover laminates and cured foam core measured in material directions ( $x_1, x_2, x_3$ ) were determined from experiments conducted in accordance with the relevant ASTM standards (1990). The material properties as well as the densities of the cover laminates and cured foam solid are given in Table 1. The laminated composite foam-filled sandwich plates were fabricated via the vacuum bag molding approach in which a stack of foam tapes was placed between two cover laminates and the whole assembly was co-cured in a vacuum. The lamina thickness of the face sheets was 0.015 cm.

The experimental apparatus for strength test of sandwich plates consisted of a 10-ton Instron testing machine, an acoustic emission (AE) system (AMS3) with two AE sensors, a displacement gauge (LVDT), two strain gauges, a data acquisition system, a steel load applicator with a spherical head (radius  $r = 0.5$  cm), and a fixture for clamping a specimen. A sketch of the experimental setup is shown in Fig. 3. The fixture was made up of two square steel frames. During testing, the laminated sandwich plate was clamped by the two steel frames which were connected together by four bolts. It is noted that the clamping method allowed no rotations at the edges of the laminated plate during loading. A stroke control approach was adopted in constructing the load-deflection relation for the laminated plate. The loading rate was slow enough for inertia effects to be neglected. During loading, the displacement gauge and data acquisition system were used to record center deflections so that the load-displacement curve of the sandwich plate could be determined. Figure 4 shows the load-displacement curves of the  $[0_3^{\circ}/0.142 \text{ cm}]_s$ ,  $[0^{\circ}/90^{\circ}/0^{\circ}/0.144 \text{ cm}]_s$  and  $[45^{\circ}/-45_2^{\circ}/0.136 \text{ cm}]_s$  sandwich plates produced by the data acquisition system. It is noted that among the sandwich plates the  $[45^{\circ}/-45_2^{\circ}/\text{core}]_s$  plate possessed the largest stiffness at the early stage of the loading process. In addition, two acoustic emission sensors were used to measure the stress waves released at the AE sources in each sandwich plate. The measured acoustic emissions were converted by the AMS3 (AE) system to a set of signal descriptors such as peak amplitude, energy, rise time and duration which were then used to identify the failure load of the sandwich plate. For instance, Fig. 5 shows the energy-applied load diagrams for the  $[45^{\circ}/-45_2^{\circ}/\text{core}]_s$  sandwich plate produced by the AMS3 system. Failure loads of the other laminated composite sandwich plates are listed in Fig. 5. Furthermore, strains along the perpendicular to fiber direction,  $\varepsilon_1$  and  $\varepsilon_2$ , respectively, at some points on the bottom surface of the  $[0^{\circ}/90^{\circ}/0^{\circ}/\text{core}]_s$  sandwich plate were measured via a strain measurement system.

#### 5. Numerical examples and discussion

Before proceeding to the minimum weight design of laminated composite sandwich plates, it is worth demonstrating the accuracy of the present finite element method in predicting strains and determining failure loads of laminated composite sandwich plates. The sandwich plates tested in the previous section were analyzed via the present finite element method using a  $4 \times 4$  mesh over a

Table 1  
Material properties of composite lamina and foam solid

Material	Material constants					Strengths (MPa)*						Density $\rho(\text{kg m}^{-3})$
	$E_1$	$E_2 = E_3$	$G_{12} = G_{13}$	$G_{23}$	$\nu_{12} = \nu_{23} = \nu_{13}$	$X_T$	$X_C$	$Y_T = Z_T$	$Y_C = Z_C$	$R$	$S = T$	
Gr/Ep	132.5 Gpa	7.90 Gpa	4.20 Gpa	1.02 Gpa	0.28	1690.75	1893.64	41.06	206.01	46.1	58.7	1450
Foam solid	1.03 Gpa	1.03 Gpa	0.4 Gpa	0.4 Gpa	0.30	6.25	10.20	6.25	10.20	3.76	3.76	267.5

\*  $R, S, T$  are shear strengths on  $x_2$ - $x_3, x_1$ - $x_2, x_1$ - $x_3$  planes, respectively.

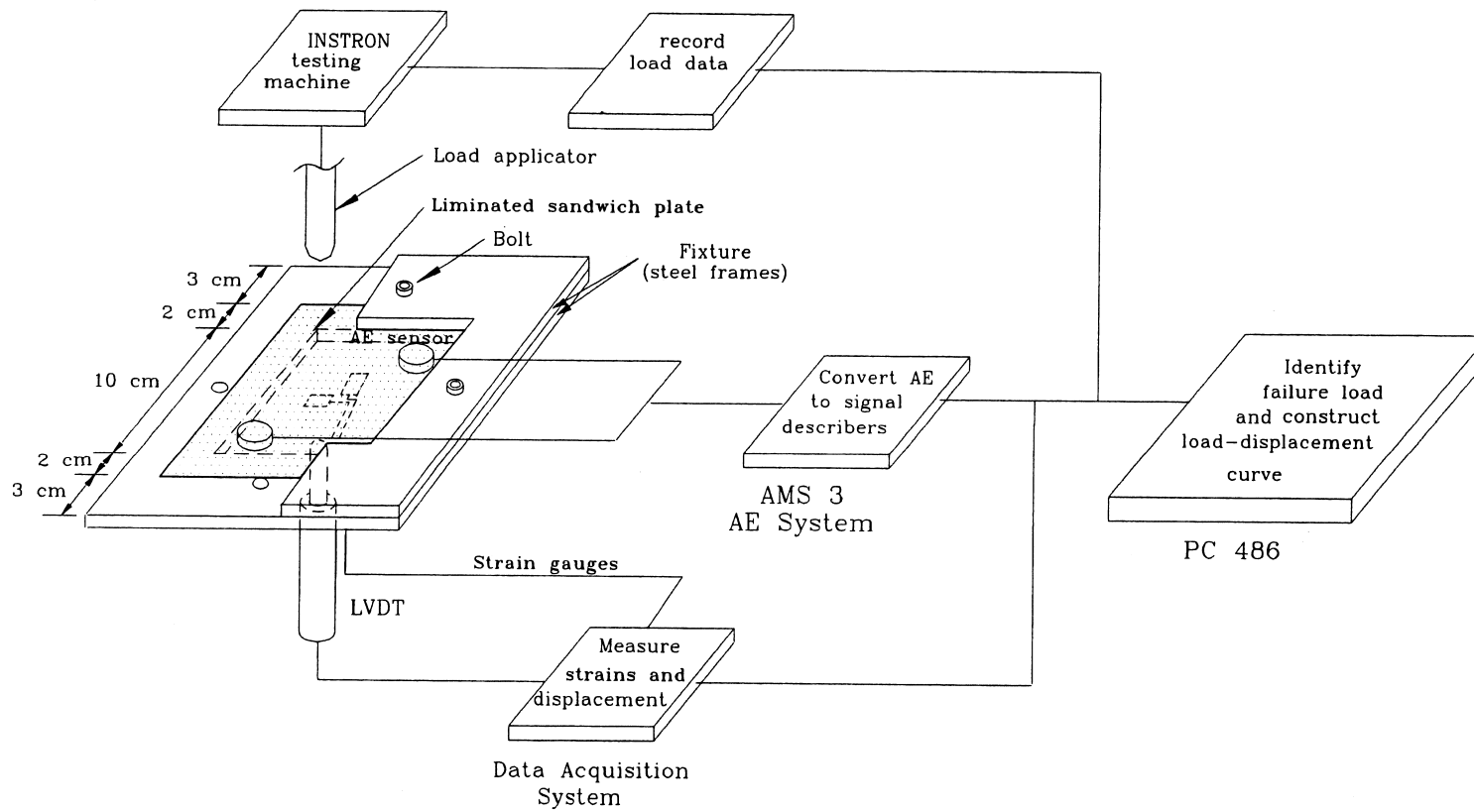


Fig. 3. A schematic description of the experimental setup.

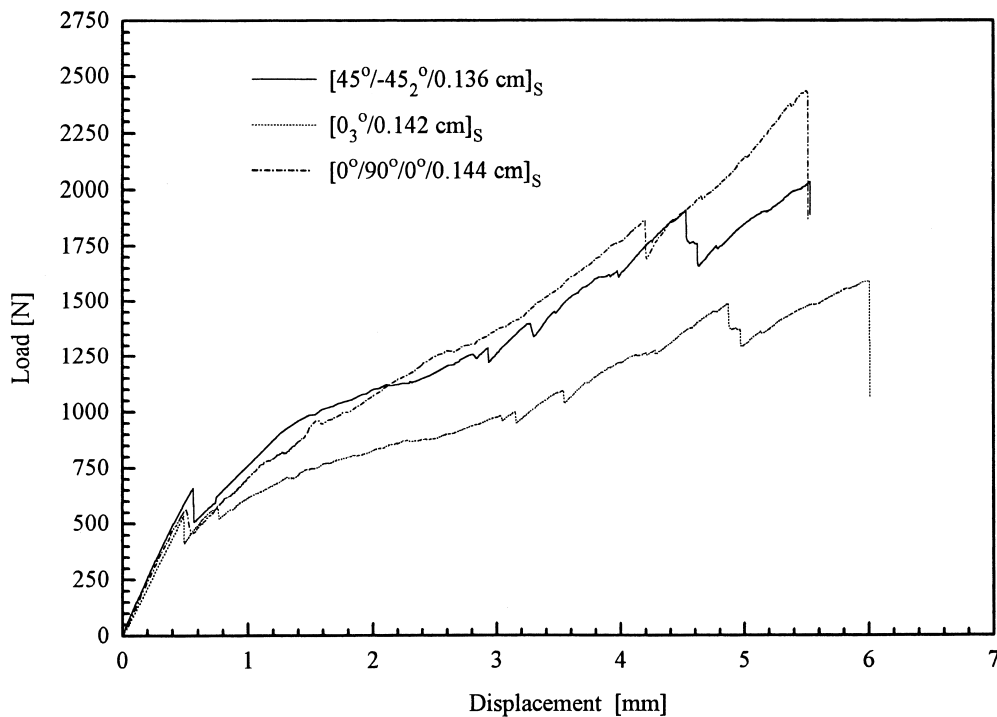


Fig. 4. Experimental load-displacement curves of sandwich plates.

full plate and number of ply groups  $NP = 7$ . For comparison purpose, the sandwich plates were also analyzed using the finite element formulated on the basis of Mindlin plate theory (Kam and Chang, 1993). In the finite element analysis, both translational and rotational degrees of freedom at the edges of the sandwich plates were constrained. Figure 6 shows the theoretical and experimental results on strains along and perpendicular to fiber direction ( $\varepsilon_1, \varepsilon_2$ ) at points with coordinates (5.0, 4.375) and (4.375, 5.0), respectively, on the bottom surface of the  $[0^\circ/90^\circ/0^\circ/\text{core}]_s$  sandwich plate. It is noted that the present finite element can predict more accurate strains than the one formulated on the basis of Mindlin plate theory when compared with the experimental results. The theoretical and experimental failure loads together with the specific strengths, which are defined as the ratios of failure loads to plate weights, are listed in Table 2 for comparison. Again for the two finite element methods, the present finite element can yield much better results when compared with the experimental results and the Mindlin type finite element in general overestimates the strengths of the sandwich plates. For instance, the present finite element method yields an error of 2.81% for the  $[0_3^\circ/0.142 \text{ cm}]_s$  plate in contrast with the error of 10.56% yielded by the Mindlin type finite element for the same case. Therefore, the present finite element method using the  $4 \times 4$  mesh will be adopted in the following optimal design of composite sandwich plates. The theoretical and experimental results show that both fiber angles and number of layer groups have significant effects on the strength of the sandwich plates. The effects of fiber angles are reflected by the fact that the specific strength of the  $[45^\circ/-45_2^\circ/0.136 \text{ cm}]_s$  plate is higher than those

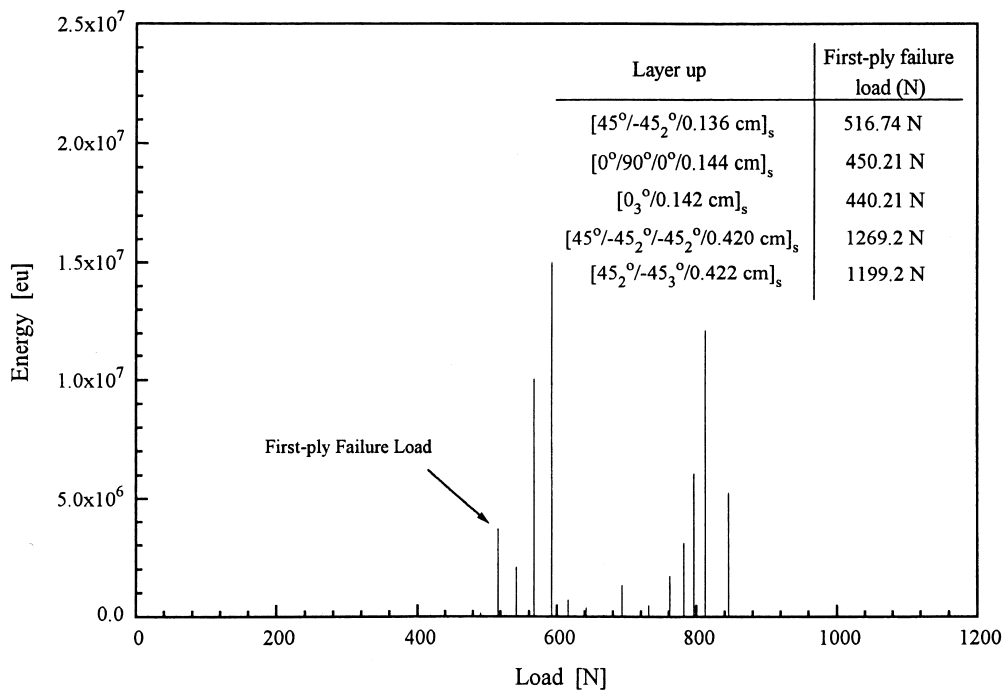


Fig. 5. Load-energy relations derived from AE system for the  $[45^\circ/-45_2^\circ/\text{core}]_s$  plate.

of the  $[0_3^\circ/0.142 \text{ cm}]_s$  and  $[0^\circ/90^\circ/0^\circ/0.144 \text{ cm}]_s$  plates while those of number of layer groups are demonstrated by the fact that the specific strength of the  $[45^\circ/-45_2^\circ/45_2^\circ/0.42 \text{ cm}]_s$  plate which has three layer groups in each face laminate is higher than that of the  $[45_2^\circ/-45_3^\circ/0.422 \text{ cm}]_s$  plate which has only two layer groups in each face laminate.

The minimum weight design of symmetrically laminated composite sandwich plates with different loading conditions, aspect ratios, numbers of layer groups and boundary conditions are performed using the material properties listed in Table 1. The boundary conditions of simple supports and clamped edges for laminated composite sandwich plates are shown in Fig. 7. The production constants of fiber angle and ply thickness for cover laminates are set as  $\theta_o = \pi/36 \text{ rad}$  (or  $5^\circ$ ) and  $h_o = 0.015 \text{ cm}$ , respectively, and the maximum allowable plate thickness is  $h^* = 2.5 \text{ cm}$ . The use of the present optimal design method is first illustrated via an example of the design of a clamped square ( $a = 10 \text{ cm}$ ) symmetrically laminated composite sandwich plate composed of seven layer groups and subjected to uniform load  $P = 420 \text{ N cm}^{-2}$ . In the first level optimization, a number of local optima including the transitional global optimum were obtained via the constrained global optimization algorithm. For comparison purposes, each of the local optimal solutions obtained in the first level optimization for the laminated composite sandwich plate was used separately in the second level optimization to determine the ‘final’ optimal solution and the results are listed in Table 3. It is noted that the use of the transitional global optimal solution in the second level optimization gives the least weight for the sandwich plate. The process of obtaining the true global optimal solution using the transitional global optimal solution as an initial guess in the second

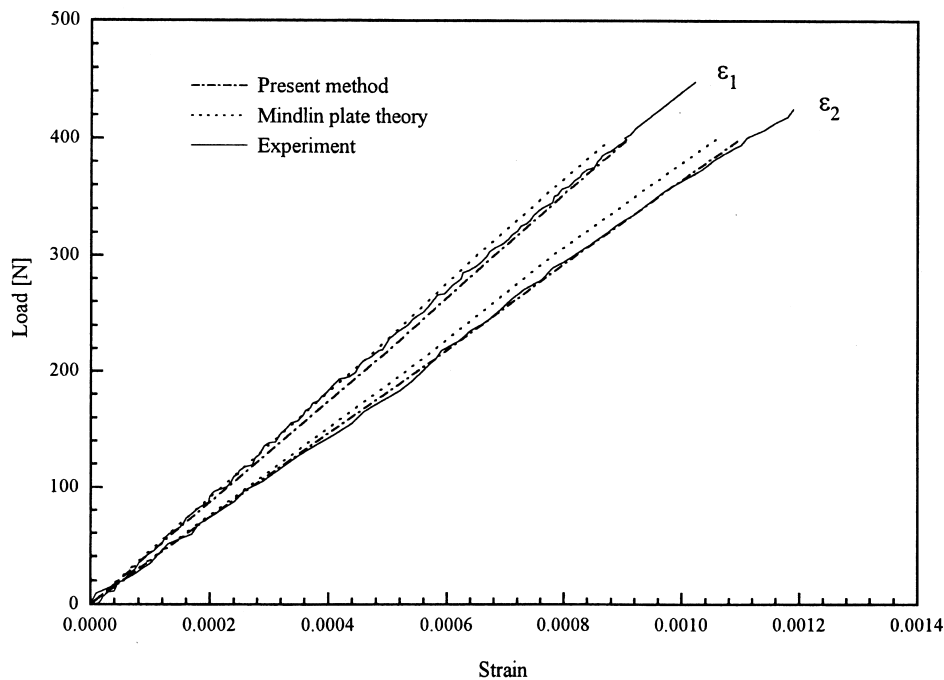


Fig. 6. Theoretical and experimental predictions of strains.

level optimization is described as follows. The layer group fiber angles are rounded off to the nearest available discrete values to give the modified transitional global optimum  $[45_{0.9}^{\circ}/45_{1.2}^{\circ}/245_{20.7}^{\circ}/0.903 \text{ cm}]_s$  where the subscript for each fiber angle denotes the number of plies in that layer group. The modified transitional global optimal solution is then used in the second level optimization to determine the true optimal layer group ply numbers and core thickness as schematically shown in Fig. 8. It is noted that the branch and bound method of Fig. 8 started the search from node 1 by treating the number of plies  $n_1$  of the first layer group as an integer variable. At node 2 an inferior design was obtained from the plate comprising less number of layer groups, i.e.,  $NL$  reduces from 7 to 5 and  $n_1 = 0$ . The true optimal solution was obtained at node 4 in which the optimal values of the design variables are  $n_1 = 1$ ,  $n_2 = 1$ ,  $n_3 = 21$ , and  $h_c = 0.898 \text{ cm}$ . If the number of plies  $n_3$  of the third layer group instead of  $n_1$  of the first layer group was treated as an integer variable at node 1, more nodes had to be fathomed before the true optimal solution could be identified at node 6 as shown in Fig. 9. It is worth pointing out that since the critical layer group, where failure is likely to occur, has the most significant effect on the convergence of the solution, the choice of the number of plies  $n_i$  of the critical layer group as an integer variable at node 1 will expedite the search process and thus reduce the number of nodes to be fathomed. In general, the true optimal solution only deviates slightly from the modified transitional optimal solution and the optimal solution can be obtained efficiently via the search process of fathoming a few nodes. The optimal solutions for the clamped and simply supported laminated composite sandwich plates with different numbers of layer groups subjected to the uniform load of intensity  $P = 420 \text{ N cm}^{-2}$  are given in Tables 4 and 5. It is noted that aspect ratio has more significant

Table 2  
Theoretical and experimental failure loads of sandwich plates

Sandwich plate	Theoretical				Experimental		Difference	
	Present method		Mindlin theory		(iii) First-ply failure load (N)	Specific strength	Difference	
	(i) $P_c$ (N)	Specific strength	(ii) $P_c$ (N)	Specific strength			$ (iii) - (i)/(iii) \%$	$ (iii) - (ii)/(iii) \%$
$[0_3^{\circ}/0.142 \text{ cm}]_s$	452.5	21.92	486.68	23.57	440.21	21.32	2.81	10.56
$[45^{\circ}/-45_2^{\circ}/0.136 \text{ cm}]_s$	508.6	25.04	578.61	28.36	516.74	25.44	1.56	11.97
$0^{\circ}/90^{\circ}/0^{\circ}/0.144 \text{ cm}]_s$	468.6	22.58	531.7	25.62	450.23	21.69	4.06	18.10
$[45_2^{\circ}/-45_3^{\circ}/0.422 \text{ cm}]_s$	1238.9	27.95	1349.4	30.44	1199.2	27.05	3.32	12.53
$[45^{\circ}/-45_2^{\circ}/45_2^{\circ}/0.420 \text{ cm}]_s$	1289.6	29.16	1408.5	31.85	1269.2	28.70	1.61	10.98

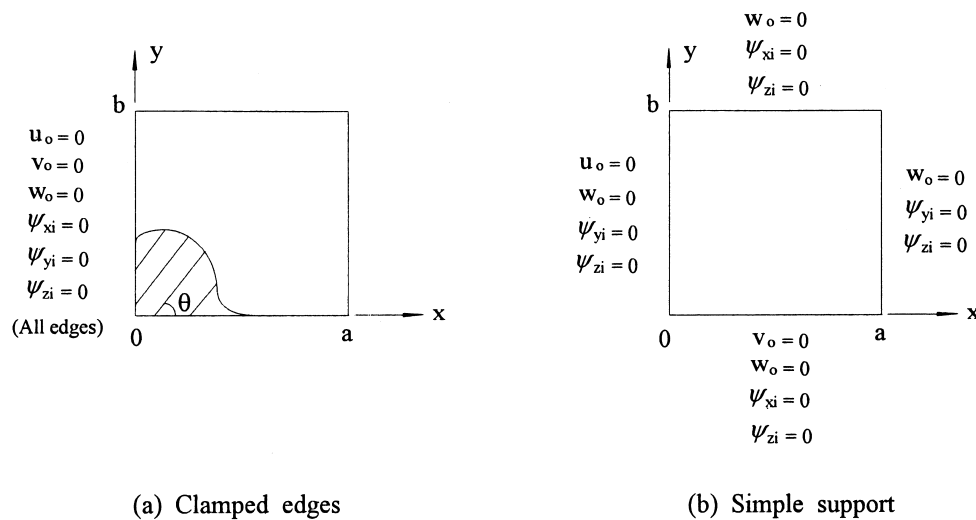


Fig. 7. Boundary conditions of sandwich plates.

effects on the optimal design parameters of the clamped sandwich plates than on those of the simply supported ones. For instance, the fiber angles of the cover laminates of the clamped plates change from  $90^\circ$  for  $b/a = 0.5$  to  $\pm 45^\circ$  for  $b/a = 1.0$  as shown in Table 4 while those of the simply supported plates change from around  $\pm 70^\circ$  to  $\pm 45^\circ$  as shown in Table 5. Irrespective to the aspect ratio and the type of boundary conditions, an increase in number of layer groups decreases the weight of the laminated composite sandwich plates. The decrease of plate weight, however, tends to slow down when the number of layer groups reaches nine. It is worth noting that the use of small number of layer groups can reduce the manufacturing time for laminated composite sandwich plates and thus make the products more economical. It is also worth noting that the differences between the transitional optimal weights at the first level optimization and the true optimal weights at the second level optimization are negligible ( $< 0.5\%$ ). This implies that the process of rounding off the transitional optimal fiber angles to the nearest integer at the second level optimization is reasonable and acceptable. Furthermore, it is also easy to realize that excellent results can still be obtained even when the final optimal ply number of layer groups are determined using the method of rounding at the second level optimization. It is noted that for any of the optimally designed sandwich plates the inner most layer groups which are adjacent to the core contain most of the plies in the plate. For instance, the fourth layer group of the  $[10_1^\circ - 25_1^\circ / -25_1^\circ / -35_1^\circ / 40_{35}^\circ / 0.693 \text{ cm}]_s$  plate in Table 5 contains 35 plies with fiber angle of  $40^\circ$ . Thus, the layup of uneven ply distribution in layer groups is generally better than that of even ply distribution. It is also interesting to study the effects on plate weight induced by the removal of the total plate thickness constraint. Table 6 lists the optimal solutions for the uniformly loaded square sandwich plates without the constraint on plate thickness. It is noted that irrespective to the boundary conditions the total number of plies in the cover sheets of any of the sandwich plates reduces drastically while the core thickness increases significantly. For instance, the number of plies in the upper cover sheet of the



Table 3  
Local optima of clamped square sandwich plate subjected to uniform load

		Optimal solution			
		1	2	3	4
First level optimization	Layup	$[30.5^\circ_{1,0}/-50.6^\circ_{1,8}/45.0^\circ_{21,2}/0.916$	$[44.8^\circ_{0,9}/-45.2^\circ_{11,9}/-45.0^\circ_{12,0}/$	$[0.5^\circ_{0,8}/90.2^\circ_{1,5}/0.2^\circ_{20,6}/$	* $[43.2^\circ_{0,9}/-45.0^\circ_{1,2}/44.8^\circ_{20,7}/$
	Weight**	cm] <sub>s</sub> 153.41	0.894 cm] <sub>s</sub> 151.36	0.912cm] <sub>s</sub> 148.42	0.903 cm] <sub>s</sub> 147.49
Second level optimization	Layup	$[30^\circ_1/-50^\circ_2/45^\circ_{21}/0.928$ cm] <sub>s</sub>	$[45^\circ_1/-45^\circ_{24}/0.890$ cm] <sub>s</sub>	$[0^\circ_1/90^\circ_2/0^\circ_{20}/0.909$ cm] <sub>s</sub>	$[45^\circ_1/-45^\circ_1/45^\circ_{21}/0.898$ cm] <sub>s</sub>
	Weight	153.74	151.84	149.01	148.09

\* Transitional global optimum; \*\* weight in gm;  $a = 10$  cm,  $P = 420$  N cm<sup>-2</sup>.

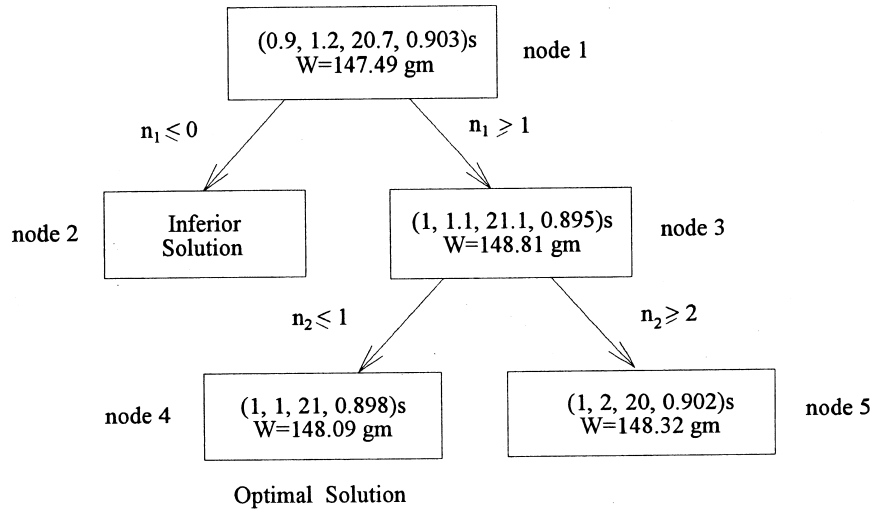


Fig. 8. Tree representation of search procedure with  $n_1$  being treated initially as an integer.

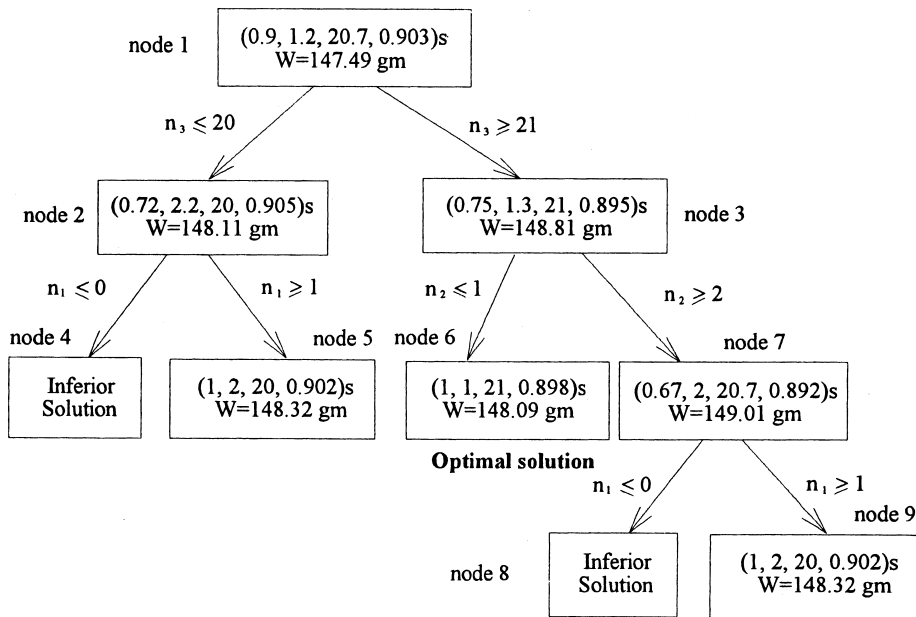


Fig. 9. Tree representation of search procedure with  $n_3$  being treated initially as an integer.

clamped square sandwich plate of  $NL = 5$  in Table 6 reduces from 24 to 8 and the plate weight gains a 25% reduction when compared with the square plate of  $NL = 5$  in Table 4.

Next, consider the optimal design of centrally loaded and symmetrically laminated composite sandwich plates. The optimal solutions for the plates with different boundary conditions, numbers

Table 4  
Optimal solutions for clamped and uniformly loaded laminated composite sandwich plates designed for minimum weight

Aspect ratio $b/a$	Number of layer groups																
	$NL = 5$					$NL = 7$					$NL = 9$						
	First level optimization		Second level optimization			Weight difference (II-I/I)%	First level optimization		Second level optimization			Weight difference (II-I/I)%	First level optimization		Second level optimization		
Solution	Weight* (I)	Solution	Weight (II)	Solution	Weight (I)		Solution	Weight (II)	Solution	Weight (I)	Solution		Weight (I)	Solution	Weight (II)	Weight difference (II-I/I)%	
0.5	$[90_{3.0}^{\circ}/90_{3.9}^{\circ}/0.974 \text{ cm}]_s$	40.99	$[90_7^{\circ}/0.970 \text{ cm}]_s$	41.10	0.27	$[90_{0.7}^{\circ}/90_{2.8}^{\circ}/90_{3.4}^{\circ}/0.974 \text{ cm}]_s$	40.99	$[90_7^{\circ}/0.970 \text{ cm}]_s$	41.10	0.27	$[90_{0.6}^{\circ}/90_{1.7}^{\circ}/90_{2.8}^{\circ}/90_{2.6}^{\circ}/0.974 \text{ cm}]_s$	40.99	$[90_7^{\circ}/0.970 \text{ cm}]_s$	41.10	0.27		
1.0	$[44.9_{0.8}^{\circ}/-45.2_{23.0}^{\circ}/0.894 \text{ cm}]_s$	151.36	$[45_1^{\circ}/-45_{23}^{\circ}/0.888 \text{ cm}]_s$	151.91	0.36	$[43.2_{0.9}^{\circ}/-45.0_{1.2}^{\circ}/44.8_{20.7}^{\circ}/0.903 \text{ cm}]_s$	147.49	$[45_1^{\circ}/-45_1^{\circ}/45_{21}^{\circ}/0.898 \text{ cm}]_s$	148.09	0.41	$[43.2_{0.9}^{\circ}/-45_{1.2}^{\circ}/43.2_{1.6}^{\circ}/-44.8_{18.1}^{\circ}/0.924 \text{ cm}]_s$	144.26	$[45_1^{\circ}/-45_1^{\circ}/45_{21}^{\circ}/-45_{18}^{\circ}/0.918 \text{ cm}]_s$	144.81	0.38		
1.2	$[23.5_{7.6}^{\circ}/-19.7_{21.2}^{\circ}/0.828 \text{ cm}]_s$	203.49	$[25_8^{\circ}/-20_{21}^{\circ}/0.821 \text{ cm}]_s$	204.09	0.29	$[-10.2_{3.1}^{\circ}/13.8_{5.5}^{\circ}/2.1_{18.2}^{\circ}/0.852 \text{ cm}]_s$	194.59	$[-10_{3}^{\circ}/15_{5}^{\circ}/0_{19}^{\circ}/0.845 \text{ cm}]_s$	195.20	0.31	$[9.2_{2.3}^{\circ}/-11.2_{5.1}^{\circ}/13.8_{5.8}^{\circ}/1.5_{13.6}^{\circ}/0.848 \text{ cm}]_s$	194.30	$[10_2^{\circ}/-10_{5}^{\circ}/15_6^{\circ}/0_{14}^{\circ}/0.843 \text{ cm}]_s$	195.13	0.42		

\* Weight in gm;  $a = 10 \text{ cm}$ ,  $P_c = 410 \text{ N cm}^{-2}$ .

Table 5  
Optimal solutions for simply supported and uniformly loaded laminated composite sandwich plates designed for minimum weight

Aspect ratio $b/a$	Number of layer groups														
	$NL = 5$					$NL = 7$					$NL = 9$				
	First level optimization		Second level optimization			Weight difference (II-I/I)%	First level optimization		Second level optimization			First level optimization		Second level optimization	
Solution	Weight* (I)	Solution	Weight (II)	Solution	Weight (I)		Solution	Weight (II)	Weight difference (II-I/I)%	Solution	Weight (I)	Solution	Weight (II)		
0.5	$[72.1_{3.7}^{\circ}/ -61.5_{3.2}^{\circ}/ 1.104 \text{ cm}]_s$	44.46	$[70_4^{\circ}/ -60_{32}^{\circ}/ 1.100 \text{ cm}]_s$	44.58	0.27	$[72.2_{0.7}^{\circ}/ -61.0_{2.3}^{\circ}/ 70.7_{3.9}^{\circ}/ 1.075 \text{ cm}]_s$	43.68	$[70_3^{\circ}/ -60_{31}^{\circ}/ 70_{31}^{\circ}/ 1.071 \text{ cm}]_s$	43.79	0.23	$[71.2_{0.7}^{\circ}/ -59.1_{1.8}^{\circ}/ 74.2_{0.9}^{\circ}/ -70.7_{2.5}^{\circ}/ 1.067 \text{ cm}]_s$	43.48	$[70_1^{\circ}/ -60_2^{\circ}/ 75_1^{\circ}/ -70_{31}^{\circ}/ 1.063 \text{ cm}]_s$	43.59	0.25
1.0	$[44.9_{2.3}^{\circ}/ -45.2_{32.5}^{\circ}/ 0.756 \text{ cm}]_s$	187.48	$[45_2^{\circ}/ -45_{32}^{\circ}/ 0.751 \text{ cm}]_s$	188.07	0.32	$[43.2_{0.8}^{\circ}/ -45.0_{1.1}^{\circ}/ 44.8_{30.9}^{\circ}/ 0.766 \text{ cm}]_s$	183.66	$[45_1^{\circ}/ -45_1^{\circ}/ 45_{31}^{\circ}/ 0.761 \text{ cm}]_s$	184.26	0.33	$[43.2_{0.8}^{\circ}/ -45.0_{1.2}^{\circ}/ 43.2_{0.7}^{\circ}/ -44.8_{29.1}^{\circ}/ 0.783 \text{ cm}]_s$	180.22	$[45_1^{\circ}/ -45_1^{\circ}/ 45_1^{\circ}/ -45_{29}^{\circ}/ 0.778 \text{ cm}]_s$	180.82	0.33
1.2	$[31.2_{1.7}^{\circ}/ -29.8_{37.1}^{\circ}/ 0.675 \text{ cm}]_s$	245.87	$[30_2^{\circ}/ -30_{37}^{\circ}/ 0.670 \text{ cm}]_s$	246.59	0.29	$[33.3_{1.2}^{\circ}/ -46.1_{0.9}^{\circ}/ -21.2_{35.7}^{\circ}/ 0.699 \text{ cm}]_s$	242.19	$[35_1^{\circ}/ -45_1^{\circ}/ -20_{36}^{\circ}/ 0.695 \text{ cm}]_s$	242.98	0.33	$[12.1_{0.9}^{\circ}/ -24.6_{0.8}^{\circ}/ -35.3_{1.2}^{\circ}/ 40.2_{34.9}^{\circ}/ 0.696 \text{ cm}]_s$	241.97	$[10_1^{\circ}/ -25_1^{\circ}/ -35_1^{\circ}/ 40_{35}^{\circ}/ 0.693 \text{ cm}]_s$	242.85	0.34

\* Weight in gm;  $a = 10 \text{ cm}$ .  $P_c = 420 \text{ N cm}^{-2}$ .

Table 6  
Optimal solutions of uniformly loaded square sandwich plates with different boundary conditions

Boundary	$NL = 5$	Weight*	$NL = 7$	Weight	$NL = 9$	Weight
Clamped edges	$[45_2/-45_6/1.475 \text{ cm}]_s$	113.71	$[45_1/-45_1/45_6/1.401 \text{ cm}]_s$	109.75	$[45_1/-45_1/45_1/-45_5/1395 \text{ cm}]_s$	119.43
Simple supports	$[45_3/-45_6/1.573 \text{ cm}]_s$	123.31	$[45_1/-45_1/45_7/1.551 \text{ cm}]_s$	122.13	$[45_1/-45_1/45_1/-45_6/1.546 \text{ cm}]_s$	121.86

\* Weight in gm;  $(a/b) = 1$ ,  $P_c = 420 \text{ N cm}^{-2}$ .

Table 7  
Optimal solutions for clamped and centrally loaded laminated composite sandwich plates designed for minimum weight

Aspect ratio $b/a$	Number of layer groups														
	$NL = 5$					$NL = 7$					$NL = 9$				
	First level optimization		Second level optimization			Weight difference (II-I/I)%	First level optimization		Second level optimization			First level optimization		Second level optimization	
Solution	Weight* (I)	Solution	Weight (II)	Solution	Weight (I)		Solution	Weight (II)	Solution	Weight (I)	Solution	Weight (II)			
0.5	$[64.3_{0.9}^{\circ}/-76.2_{6.0}^{\circ}/0.708 \text{ cm}]_s$	33.89	$[65_{1}^{\circ}/-75_{6}^{\circ}/0.703 \text{ cm}]_s$	33.98	0.27	$[-59.7_{0.8}^{\circ}/65.1_{0.9}^{\circ}/-74.8_{3.2}^{\circ}/0.697 \text{ cm}]_s$	33.60	$[-60_{1}^{\circ}/65_{1}^{\circ}/-75_{3}^{\circ}/0.692 \text{ cm}]_s$	33.69	0.27	$[-58.9_{0.8}^{\circ}/63.7_{1.0}^{\circ}/-74.1_{3.8}^{\circ}/-75.4_{2.2}^{\circ}/0.697 \text{ cm}]_s$	33.60	$[-60_{1}^{\circ}/65_{1}^{\circ}/-75_{3}^{\circ}/0.692 \text{ cm}]_s$	31.51	0.27
1.0	$[44.9_{1.8}^{\circ}/-45.0_{3.1}^{\circ}/0.565 \text{ cm}]_s$	51.54	$[45_{2}^{\circ}/-45_{3}^{\circ}/0.561 \text{ cm}]_s$	51.76	0.43	$[45.1_{0.9}^{\circ}/-45_{1.5}^{\circ}/45.0_{2.5}^{\circ}/0.557 \text{ cm}]_s$	51.11	$[45_{1}^{\circ}/-45_{3}^{\circ}/45_{3}^{\circ}/0.554 \text{ cm}]_s$	51.39	0.55	$[45.0_{0.6}^{\circ}/-45.0_{0.8}^{\circ}/45.0_{1.5}^{\circ}/-44.8_{2.1}^{\circ}/0.551 \text{ cm}]_s$	51.22	$[45_{1}^{\circ}/-45_{1}^{\circ}/45_{1}^{\circ}/-45_{1}^{\circ}/0.552 \text{ cm}]_s$	51.28	0.11
1.2	$[48.6_{0.9}^{\circ}/-14.5_{3.1}^{\circ}/0.540 \text{ cm}]_s$	60.77	$[50_{1}^{\circ}/-15_{4}^{\circ}/0.541 \text{ cm}]_s$	60.83	0.11	$[14.6_{0.6}^{\circ}/-47.9_{0.7}^{\circ}/15.2_{2.6}^{\circ}/0.544 \text{ cm}]_s$	60.50	$[15_{1}^{\circ}/-50_{1}^{\circ}/15_{3}^{\circ}/0.538 \text{ cm}]_s$	60.64	0.23	$[14.1_{0.7}^{\circ}/-49.9_{1.0}^{\circ}/14.8_{3.1}^{\circ}/15.2_{1.1}^{\circ}/0.540 \text{ cm}]_s$	60.48	$[15_{1}^{\circ}/-50_{1}^{\circ}/15_{3}^{\circ}/0.538 \text{ cm}]_s$	60.64	0.27

\* Weight in gm;  $a = 10 \text{ cm}$ ,  $P_c = 2000 \text{ N}$ .

Table 8  
Optimal solutions for simply supported and centrally loaded laminated composite sandwich plates designed for minimum weight

Aspect ratio $b/a$	Number of layer groups															
	$NL = 5$					$NL = 7$					$NL = 9$					
	First level optimization		Second level optimization			Weight difference (II-I/I)%	First level optimization		Second level optimization			Weight difference (II-I/I)%	First level optimization		Second level optimization	
Solution	Weight* (I)	Solution	Weight (II)	Solution	Weight (I)		Solution	Weight (II)	Weight difference (II-I/I)%	Solution	Weight (I)		Solution	Weight (II)	Weight difference (II-I/I)%	
0.5	[59.2 <sub>0.8</sub> <sup>o</sup> / -72.6 <sub>0.1</sub> <sup>o</sup> / 0.732 cm] <sub>s</sub>	34.54	[60 <sub>1</sub> <sup>o</sup> / -75 <sub>6</sub> <sup>o</sup> / 0.726 cm] <sub>s</sub>	34.59	0.14	[-60.2 <sub>0.6</sub> <sup>o</sup> / 58.3 <sub>0.8</sub> <sup>o</sup> / -76.7 <sub>5.5</sub> <sup>o</sup> / 0.719 cm] <sub>s</sub>	34.19	[-60 <sub>1</sub> <sup>o</sup> / 60 <sub>1</sub> <sup>o</sup> / -75 <sub>5</sub> <sup>o</sup> / 0.714 cm] <sub>s</sub>	34.43	0.70	[-59.9 <sub>0.6</sub> <sup>o</sup> / 59.5 <sub>0.8</sub> <sup>o</sup> / -74.9 <sub>2.4</sub> <sup>o</sup> / -76.2 <sub>3.1</sub> <sup>o</sup> / 0.719 cm] <sub>s</sub>	34.19	[-60 <sub>1</sub> <sup>o</sup> / 60 <sub>1</sub> <sup>o</sup> / -75 <sub>5</sub> <sup>o</sup> / 0.714 cm] <sub>s</sub>	34.43	0.70	
1.0	[45.1 <sub>2.0</sub> <sup>o</sup> / -45.0 <sub>3.9</sub> <sup>o</sup> / 0.572 cm] <sub>s</sub>	56.27	[45 <sub>2</sub> <sup>o</sup> / -45 <sub>4</sub> <sup>o</sup> / 0.568 cm] <sub>s</sub>	56.49	0.40	[44.9 <sub>0.8</sub> <sup>o</sup> / -45.0 <sub>1.6</sub> <sup>o</sup> / 45.1 <sub>3.5</sub> <sup>o</sup> / 0.560 cm] <sub>s</sub>	55.63	[45 <sub>1</sub> <sup>o</sup> / -45 <sub>2</sub> <sup>o</sup> / 45 <sub>3</sub> <sup>o</sup> / 0.556 cm] <sub>s</sub>	55.85	0.41	[44.5 <sub>0.8</sub> <sup>o</sup> / -45.0 <sub>0.9</sub> <sup>o</sup> / 45.2 <sub>1.5</sub> <sup>o</sup> / -44.6 <sub>2.7</sub> <sup>o</sup> / 0.557 cm] <sub>s</sub>	55.60	[45 <sub>1</sub> <sup>o</sup> / -45 <sub>1</sub> <sup>o</sup> / 45 <sub>2</sub> <sup>o</sup> / -45 <sub>2</sub> <sup>o</sup> / 0.554 cm] <sub>s</sub>	55.74	0.25	
1.2	[48.9 <sub>0.8</sub> <sup>o</sup> / -14.9 <sub>3.1</sub> <sup>o</sup> / 0.567 cm] <sub>s</sub>	67.20	[50 <sub>1</sub> <sup>o</sup> / -15 <sub>1</sub> <sup>o</sup> / 0.561 cm] <sub>s</sub>	67.33	0.19	[15.2 <sub>0.8</sub> <sup>o</sup> / -50.6 <sub>1.2</sub> <sup>o</sup> / 24.9 <sub>3.9</sub> <sup>o</sup> / 0.551 cm] <sub>s</sub>	66.17	[15 <sub>1</sub> <sup>o</sup> / -50 <sub>1</sub> <sup>o</sup> / -50 <sub>1</sub> <sup>o</sup> / 25 <sub>3</sub> <sup>o</sup> / 0.546 cm] <sub>s</sub>	66.37	0.30	[15.6 <sub>0.8</sub> <sup>o</sup> / -49.6 <sub>0.9</sub> <sup>o</sup> / 22.5 <sub>1.6</sub> <sup>o</sup> / 26.2 <sub>2.6</sub> <sup>o</sup> / 0.548 cm] <sub>s</sub>	65.98	[15 <sub>1</sub> <sup>o</sup> / -50 <sub>1</sub> <sup>o</sup> / 20 <sub>1</sub> <sup>o</sup> / 25 <sub>3</sub> <sup>o</sup> / 0.543 cm] <sub>s</sub>	66.18	0.31	

\* Weight in gm;  $a = 10$  cm,  $P_c = 2000$  N.

of layer groups and aspect ratios subjected to  $P = 2000$  N are listed in Tables 7 and 8. It is noted that most of the features observed previously for the case of uniform load are still valid for the present case. The optimal weight of the centrally loaded sandwich plates generally converges at  $NL = 9$ . The inmost layer groups of the cover sheets of the plates contain relatively large number of plies comparing to the other layer groups. The differences between the transitional optimal weights obtained at the first level optimization and the true optimal weights at the second level optimization are negligible. Again this implies that the use of the method of rounding in determining the final optimal design variables at the second level optimization is viable and can give excellent results. Unlike the case of uniform load, the adopted boundary conditions only have slight effects on the layups and weights of the centrally loaded sandwich plates. It is also noted that the theoretical optimal layups for clamped square laminated composite sandwich plates being the combination of  $\pm 45^\circ$  plies has been validated by the experimental results which have indeed shown that the specific strength of the  $[45^\circ/-45_2^\circ/0.136 \text{ cm}]_s$  plate is higher than those of the  $[0_3^\circ/0.142 \text{ cm}]_s$  and  $[0^\circ/90^\circ/0^\circ/0.144 \text{ cm}]_s$  plates. Furthermore, the experimental results of the  $[45_2^\circ/45_3^\circ/0.422 \text{ cm}]_s$  and  $[45^\circ - 45_2^\circ/45_2^\circ/0.420 \text{ cm}]_s$  sandwich plates have also verified the fact that an increase in the number of layer groups of a sandwich plate increases the strength of the plate or conversely reduces the plate weight for achieving the same strength.

## 6. Conclusion

A two level optimization method was presented to study the minimum weight design of laminated composite sandwich plates subject to strength and side constraints. The effectiveness of the proposed method in determining the optimal discrete values of layer group thicknesses and fiber angles was demonstrated via a number of examples of the design of uniformly loaded and symmetrically laminated composite sandwich plates. Experiments of centrally loaded and symmetrically laminated composite sandwich plates were performed to further verify the accuracy of the present method. Effects of loading condition, boundary condition, core thickness, aspect ratio and number of layer groups on the optimal solution of laminated composite sandwich plates were studied via a number of examples. It was found that all of the above parameters might have effects to certain degrees on the optimal solution of the plates. In particular, without altering the strength of a laminated composite sandwich plate, an increase in core thickness could reduce the cover sheet thicknesses and the plate weight significantly. Without altering the total number of plies in the sandwich plate, an increase in layer group number could also reduce the plate weight. The use of the present method in designing laminated composite sandwich plates could greatly reduce the number of layer groups and thus shorten manufacturing time. It was also shown that the optimal design process could be greatly simplified while excellent results could still be obtainable with the use of the method of rounding at the second level of optimization. In this study, it was found that the plate weight tended to converge as the number of layer groups reached nine. The experimental and theoretical results presented in this paper are useful for problem verification as well as practical applications.

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